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Comment on Resummation of Mass Distribution for Jets Initiated by Massive Quarks

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Abstract

We compute in the heavy quark effective theory the soft coefficient D_2 entering the resummation of next-to-next-to-leading threshold logarithms for jets initiated by a quark with a small mass compared to the hard scale of the process. We find complete agreement with a previous computation in full QCD. Contrary to our previous guess, this coefficient turns out to be different from that one entering heavy flavor decay or heavy flavor fragmentation.

In [1] we have considered the resummation of threshold logarithms for jets initiated by a quark with a small mass compared to the hard scale of the process. The main result is the following factorization formula, in N -moment space, for the probability that a quark with virtuality Q and mass $m \ll Q$ fragments into a jet of mass m_X :

$$J_N(Q^2; m^2) = J_N(Q^2) \delta_N(Q^2; m^2), \quad (1)$$

where $J_N(Q^2)$ is the standard massless fragmentation function, whose resummed expression reads [2, 3, 4]:

$$J_N(Q^2) = \exp \int_0^1 dy \frac{(1-y)^{N-1} - 1}{y} \left\{ \int_{Q^2 y^2}^{Q^2 y} \frac{dk_\perp^2}{k_\perp^2} A[\alpha_S(k_\perp^2)] + B[\alpha_S(Q^2 y)] \right\}, \quad (2)$$

and $\delta_N(Q^2; m^2)$ is the mass correction factor, whose resummed expression reads for $N \gtrsim 1/r$:

$$\delta_N(Q^2; m^2) = \exp \int_0^1 dy \frac{(1-y)^{r(N-1)} - 1}{y} \left\{ - \int_{m^2 y^2}^{m^2 y} \frac{dk_\perp^2}{k_\perp^2} A[\alpha_S(k_\perp^2)] - B[\alpha_S(m^2 y)] + D[\alpha_S(m^2 y^2)] \right\}. \quad (3)$$

The functions $A(\alpha_S)$, $B(\alpha_S)$ and $D(\alpha_S)$, entering the above resummation formulae, have a perturbative expansion in powers of α_S :

$$A(\alpha_S) = \sum_{n=1}^{\infty} A_n \alpha_S^n; \quad B(\alpha_S) = \sum_{n=1}^{\infty} B_n \alpha_S^n; \quad D(\alpha_S) = \sum_{n=1}^{\infty} D_n \alpha_S^n, \quad (4)$$

and we have defined¹:

$$y \equiv \frac{m_X^2 - m^2}{Q^2 - m^2}; \quad r \equiv \frac{m^2}{Q^2} \cong \frac{m^2}{Q^2 - m^2} \ll 1. \quad (5)$$

Since the functions $A(\alpha_S)$ and $B(\alpha_S)$ are related to small-angle emission only, they represent universal intra-jet properties, as confirmed by explicit higher-order computations; at present, the first three coefficients of both these functions are analytically known [6], allowing for next-to-next-to-leading logarithmic accuracy. On the contrary, the function $D(\alpha_S)$, being related to soft emission at large angle with respect to the quark, is in general a process-dependent inter-jet quantity. In the framework of fragmentation functions, soft radiation not collinearly enhanced is described by the function $D^{(f)}(\alpha_S)$ [4], while in heavy flavor decay it is described by the function $D^{(h)}(\alpha_S)$, which has been shown to coincide with the former one to all orders in α_S in [7]. In [1]

¹ The case of a non-relativistic motion of a massive color charge, $m \approx \mathcal{O}(Q)$, which does not give rise to a jet, has been analyzed in [5].

we explicitly computed the coefficient D_1 , which turned out to be equal to $D_1^{(f,h)}$. On the basis of a physical argument, we then conjectured that the equality could extend to higher orders, i.e. that $D(\alpha_S) = D^{(f,h)}(\alpha_S)$. After the publication of [1], the coefficient D_2 was computed in full QCD in [8] by using explicit results on massive two-loop vertex functions [9]. This computation showed explicitly that $D_2 \neq D_2^{(f,h)}$, invalidating our previous guess $D(\alpha) = D^{(f,h)}(\alpha)$. In this note we present an independent recomputation of D_2 in the heavy quark effective theory, based on an expansion of the two-loop cusp anomalous dimension Γ_{cusp} [10]. As in [1], let us consider the case of the $b \rightarrow s \gamma$ decay, in the rest frame of the b -quark. The hard scale of the process is given by the beauty mass, $Q = m_b$, and the mass correction parameter by $r = m_s^2/m_b^2 \ll 1$. The Minkowskian cusp angle then reads

$$\gamma = \text{arccosh} (v_b \cdot v_s) = \frac{1}{2} \log \frac{1}{r}, \quad (6)$$

where v_b and v_s are the b and s quark 4-velocities ($v_b^2 = v_s^2 = 1$). By expanding for $r \ll 1$ (two times) the cusp anomalous dimension evaluated in [10], one obtains:

$$2 \Gamma_{\text{cusp}}(r; \alpha_S) \cong [A_1 \alpha_S + A_2 \alpha_S^2] \log \frac{1}{r} + [D_1 + D_1^{(h)}] \alpha_S + [D_2 + D_2^{(h)}] \alpha_S^2, \quad (7)$$

where

$$D_1 = D_1^{(h)} = -\frac{C_F}{\pi}; \quad (8)$$

$$D_2 + D_2^{(h)} = \frac{C_F}{\pi^2} \left[C_A \left(z(2) - z(3) - \frac{49}{18} \right) - \frac{5}{18} n_f \right]. \quad (9)$$

Let us stress that we have two different D -like contributions to the cusp anomalous dimension: $D^{(h)}(\alpha)$, related to soft emission off the decaying b -quark, and $D(\alpha)$, related to the soft emission off the massive s -quark originating the jet in the final state². Fermionic contributions, not evaluated in [10], have been added by noting that they are of the form $\alpha_S^2 n_f (\log 1/r - 2)$ and by including in the coefficient A_2 above also the known $\mathcal{O}(n_f)$ contribution. By assuming that the eikonal result above can be directly related to full QCD and by subtracting from the sum above the standard $D_2^{(h)}$ for heavy flavor decay,

$$D_2^{(h)} = \frac{C_F}{\pi^2} \left[C_A \left(-\frac{9}{4} z(3) + \frac{1}{2} z(2) + \frac{55}{108} \right) + \frac{n_f}{54} \right] \simeq -0.556416 + 0.002502 n_f, \quad (10)$$

we obtain for the coefficient entering the jet-correction factor $\delta_N(Q^2, m^2)$:

$$D_2 = \frac{C_F}{\pi^2} \left[C_A \left(\frac{5}{4} z(3) + \frac{1}{2} z(2) - \frac{349}{108} \right) + \frac{29}{54} n_f \right] \simeq -0.367368 + 0.072551 n_f. \quad (11)$$

² There is no B -like contribution to the cusp anomalous dimension because the heavy quark effective theory only describes soft interactions.

The above expression for D_2 is in complete agreement with that one obtained in full QCD in Eq. (68) of [8], both in the Abelian and non-Abelian pieces (after a trivial difference in normalization is taken into account). As a consequence, the expressions for the NNLL coefficients H_{21} and H_{32} given in Eqs. (141) and (144) of [1] respectively are also incorrect and must be replaced by:

$$\begin{aligned} H_{21} &= \frac{C_F^2}{2\pi^2} \left(-\frac{3}{16} + \frac{\pi^2}{6} - 5z(3) \right) + \frac{C_F C_A}{4\pi^2} \left(5z(3) + \frac{5\pi^2}{18} - \frac{121}{72} \right) + \\ &+ \frac{C_F n_f}{\pi^2} \left(\frac{5}{144} - \frac{\pi^2}{36} \right); \end{aligned} \quad (12)$$

$$\begin{aligned} H_{32} &= -\frac{C_F^3}{4\pi^3} \left(\frac{\pi^4}{90} + z(3) \right) + \frac{C_A C_F^2}{4\pi^3} \left(-\frac{11}{32} - \frac{767\pi^2}{432} + \frac{\pi^4}{18} - 22z(3) \right) + \\ &+ \frac{C_A^2 C_F}{4\pi^3} \left(\frac{44539}{2592} - \frac{145\pi^2}{216} + \frac{11\pi^4}{360} + \frac{11z(3)}{12} \right) + \frac{n_f^2 C_F}{4\pi^3} \left(\frac{205}{648} + \frac{\pi^2}{54} \right) + \\ &+ \frac{n_f C_F^2}{4\pi^3} \left(-\frac{67}{48} + \frac{61\pi^2}{216} + 5z(3) \right) + \frac{C_A n_f C_F}{4\pi^3} \left(-\frac{6745}{1296} - \frac{7z(3)}{6} \right). \end{aligned} \quad (13)$$

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